21A3, Cyclotrons 2004, Tokyo, 18-22 October 2004

RESULTS OF 3D BEAM DYNAMIC STUDIES IN DISTORTED FIELDS OF A 250 MeV SUPERCONDUCTING CYCLOTRON

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Abstract

For PSI's new medical cyclotron *COMET*, three dimensional particle tracking calculations are performed with the code *TRACK*. We discuss some examples, such as the effects of coil misalignment and of crossing betatron resonances. Also the effect of the magnetic field from the Dee-currents is described.

INTRODUCTION

A 250 MeV SC compact cyclotron (original design by H. Blosser, NSCL, USA and manufactured by ACCEL, D) is ready for tests on site. The cyclotron, *COMET*, will provide beam for the PSI proton therapy project PROSCAN. For this cyclotron, three dimensional particle tracking calculations have been performed with the code *TRACK* [1], a general purpose particle tracking program in combined static and alternating magnetic and electric fields. Input fields are obtained from calculations or field measurements. The purposes of the calculations are an independent verification of the shimming procedures proposed and/or performed by ACCEL and to prepare for the commissioning of the cyclotron. Here we report on an investigation of the effects on particle trajectories due to several possible distortions of the magnetic field.

MEDIAN-PLANE ERROR

Position of the median plane

In several SC-cyclotrons, it appears that the vertical position of the main coil has a very big impact on beam losses in the vertical plane [e.g. 2,3]. A vertical shift of the coil leads to a shift of the magnetic median plane. This is, however, very difficult to determine with field measurements. Simulations may therefore be helpful.

The effect of the coil position is obtained by assuming that the magnetic field of *COMET* can be split into one contribution from the iron and one from the coil. We have extracted these two parts from the model, from which the magnetic field is calculated with *TOSCA*, obtained from ACCEL. For this purpose, we have recalculated the model with 1% higher current. After subtracting the old field (100% current) from the new field and multiplying the resulting field with 100 we "extracted" the coil field contribution. The difference between the total field and the coil field is then assumed to be the iron contribution. The two field contributions were saved in separate 3D-field maps and can be added with any desired relative spatial shift and/or tilt. The procedure described here, is not absolutely correct, but it is valid for small coil displacements. A practical alternative is currently computationally virtually impossible, as we would need to calculate full models and abandon the 8-fold symmetry.

From the azimuthally averaged fields derived with the method explained above, the radial field components (B_r) of the two contributions have been calculated. Up to radius r=78 cm, $\langle B_r \rangle$ is mainly coming from the coil (a few mT), so that a vertical shift δ of the coil gives an approximately equal shift of the median plane.

An analytical expression of the median plane position can be derived by inserting the median plane condition B_r=0 in the expression which describes the radial component of the field B_r (averaged over the azimuth):

$$B_r(r,z) \approx z \cdot \frac{\partial B_r^{iron}}{\partial z} + (z-\delta) \cdot \frac{\partial B_r^{coil}}{\partial z} \qquad (1)$$

Using $\partial B_r / \partial z = \partial B_z / \partial r$, the vertical position of the magnetic median plane is then described as:

$$Z(r) = \delta \cdot \frac{\partial B_z^{coil}}{\partial r} \Bigg/ \left[\frac{\partial B_z^{iron}}{\partial r} + \frac{\partial B_z^{coil}}{\partial r} \right] \qquad (2)$$

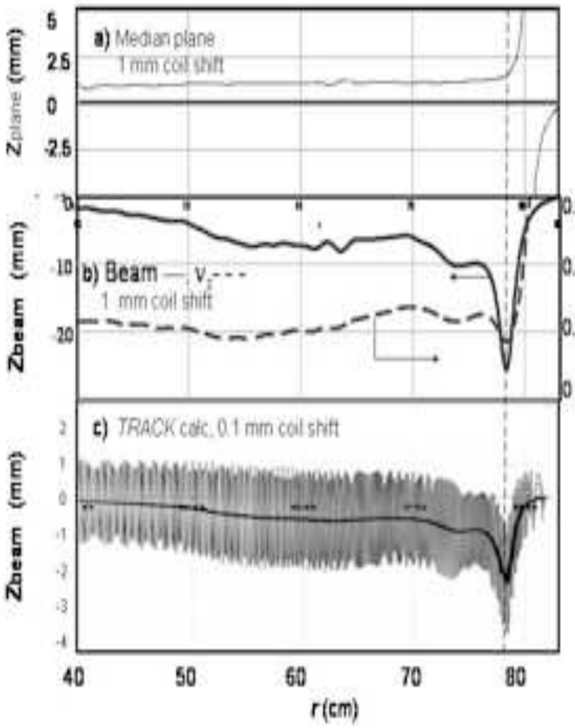

Figure 1: a) Median plane position with a coil shift δ=1 mm. b) beam position from eq (4) and ν_z , c) *TRACK* results with δ=0.1 mm for different particle starting conditions. The dashed line at r=78 cm is at minimum ν_z.

It can be convenient to rewrite eq. (2) in terms of ν_z, the vertical betatron oscillation frequency:

$$Z(r) = \frac{\partial B_z^{coil}}{\partial r} \cdot \frac{r}{B_z} . \frac{-\delta}{\nu_z^2 - \widetilde{F}} \tag{3}$$

Note that we have not neglected the flutter dependent terms ($\widetilde{F}$). We found an excellent agreement of these equations with the above mentioned addition of partial fields from *TOSCA*. In fig. 1a the median plane is shown for δ=1 mm, calculated with eq. (2) or (3). The denominators of eq. (2) and (3) become zero at r=80.3 cm, which is the radius where the field lines change their radial curvature sign ($R_{extraction}$≈82 cm). Here a median plane does not exist.

Effects on the beam

A particle starting at z=0 (i.e. the "iron" median plane), experiences a vertical force $m\ddot{z} = B_r q \omega_{cycl} r$ from the radial component of the coil field, in the direction opposite to the coil shift. The vertical focussing due to the azimuthally varying iron field pushes the particle back to z=0 and an oscillation will start. Approximated as a harmonic oscillation, the vertical position z satisfies: $z_{beam} = -\ddot{z} / \omega_z^2$, or:

$$z_{beam}(r) = -\frac{r}{B_{tot}} \frac{\partial B_z^{coil}}{\partial r} . \frac{1}{\nu_z^2} . \delta \tag{4}$$

In fig. 1b the beam position z_{beam} is shown, which is equal to the formula given in [2].

Using *TRACK*, we simulated the beam behaviour due to a median-plane shift of δ=0.1 mm. In the vertical direction, we see the same strong effect (fig. 1c) as the analytical calculation shown in fig. 1b. The beam does not "follow" the median plane (eq. 2), but a slow shift of the beam position is observed in the opposite direction with respect to the coil shift. The magnitude of the beam shift strongly depends on $1/\nu_z^2$, the focussing power coming mainly from the iron field. Although the maximum shift of the beam is approximately 25 times larger than the vertical coil displacement itself, no increase of beam size is observed. The beam's energy gain per turn and the radius increase per turn are only weakly influenced by a vertical coil shift, as can be expected.

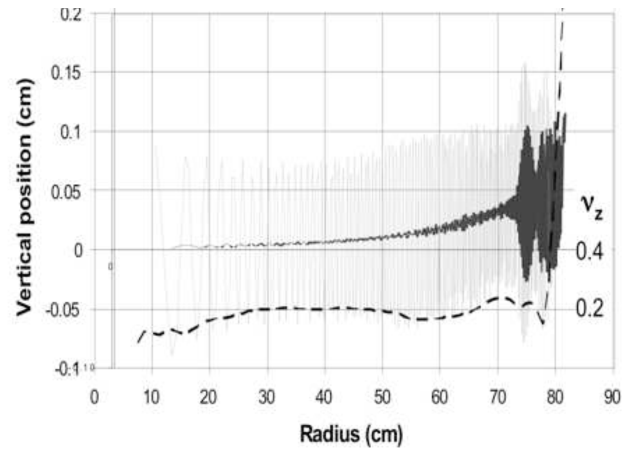

Figure 2: Vertical particle position as a function of radius when the coil is tilted 0.38 mrad. Dark line: particle starts at centre of iron gap, grey line: particle starts 1 mm above centre. Dashed line: ν_z.

Fig. 2 shows the effect of a tilted coil on the vertical beam position. The beam plane also tilts and around r=75 cm and r=78 cm strong vertical oscillations start, just where ν_z has its minima. An important effect of the coil tilt is an azimuthally varying field component (1st harmonic), which might lead to the excitation of resonances.

BETATRON RESONANCES

From the field map obtained from the *TOSCA* model or from measurements, we derived the equilibrium orbits and the frequencies of the betatron oscillations ν_r and ν_z. Fig. 3 shows the $\nu_z(\nu_r)$ plot obtained, together with some possible betatron resonances. Since especially the coupling resonance ν_r-2ν_z=0 is notorious for causing beam losses, we used *TRACK* to investigate the "width" and the effect of this resonance. First we mapped the equilibrium orbits between 244 and 247 MeV, with a spacing of 1 mm in radius. Around each equilibrium orbit several rays were tracked. These rays had initial offsets of 1 mm or 1 mrad in radial and/or vertical direction. We tracked these rays for 10 turns and recorded their deviation from the equilibrium orbit as a function of the distance covered. In fig. 4 an example is shown of a particle near the 246 MeV equilibrium orbit. For tracks near 246 MeV we could observe increasing or beating amplitudes, both in the radial and in the vertical direction, which suggests the excitation of the coupling resonance.

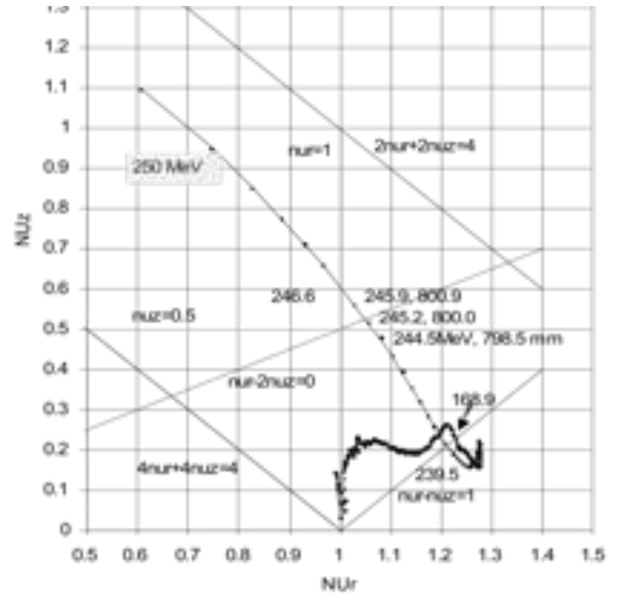

Figure 3: Plot of the betratron frequencies and resonances in the measured field of COMET. Some key energies and average orbit radii are indicated.

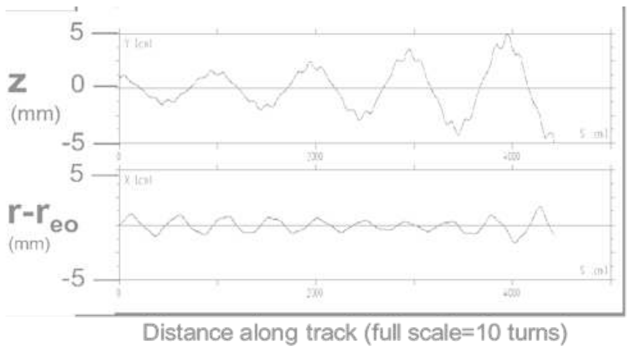

Figure 4: Track of a particle relative to the 246 MeV equilibrium orbit in the vertical plane (top) and in the horizontal plane (bottom).

The effect of the resonance also depends on how many turns the accelerated beam experiences its effect. We found that tracks showing amplitude increases of >150%, occur only between an average equilibrium-orbit radius of 800 mm and 803 mm. This corresponds to an energy gain of 1.7 MeV, or about 4 turns. The tracks such as shown in fig. 4, all indicate that an amplitude increase to ~150% builds up in approximately 5-7 turns. It can therefore be expected that the accelerated beam passes fast enough through this resonance.

To test this, we also performed tracking with the acceleration field switched on. First, a reference track starting at 239 MeV was searched, and then 8 rays with a slight change in starting conditions were traced. Fig. 5 shows the deviation of one of these rays with respect to the reference track. At approximately 246 MeV, one observes an increase in radial amplitude and a decrease in vertical amplitude. It is remarkable, that we have not observed an amplitude increase in the vertical direction in any of the accelerated particles. Although for those rays tracked, the amplitude change is about the expected value, more tracking needs to be performed to obtain a complete view of the consequences of the resonance crossing.

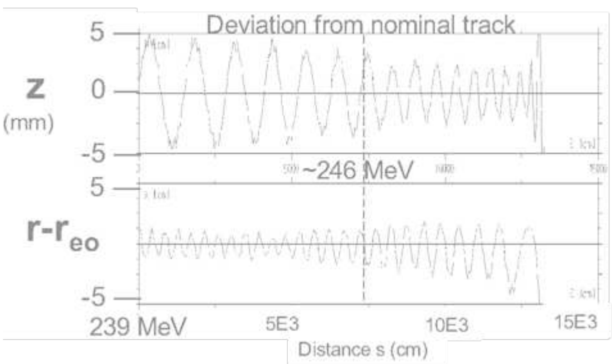

Figure 5: Track of an accelerated particle relative to the reference track starting at 239 MeV. The vertical deviation (top) and the horizontal deviation (bottom) are shown as a function of covered distance. The horizontal scale represents a distance covering ~30 turns.

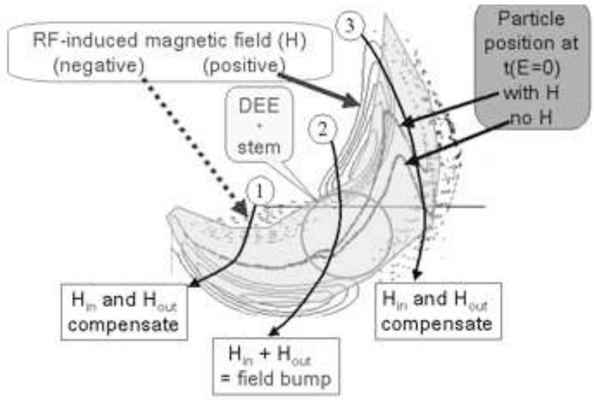

Figure 6: Three numbered particle tracks crossing the Dee and its H(t) field. The particle positions at t(E=0) with and without H(t) field are shown.

EFFECT OF CURRENTS IN THE DEE

Apart from the static magnetic field, the beam also experiences a time dependent magnetic field H(t), caused by the HF currents (de)charging the Dees and calculated with the code *Ω3p*. With *TRACK,* we marked the particle positions at the time the electric field equals zero and compared the particle positions (i.e. isochrony) for a field with and without H(t). In figure 6 it can be seen that the H-field results in a net kick of the phase of the particles, which grows to approximately 20 degrees (HF phase). This kick starts at the radius where the DEE-stems are located. Here (track 2) the H-fields at entrance and exit of the Dee do not cancel, which results in a small field bump of approximately 0.3×10^{-3} Tm. The H-fields at entrance and exit of the Dee compensate each other for tracks at smaller (track 1) or larger (track 3) radii. The phase shift $d(\sin(\phi))$ due to the H-field experienced in 200 turns is ~15 degrees at the end of the bump. At larger radii this phase shift will then grow to the earlier observed 20 degrees.

CONCLUSIONS

Three-dimensional particle tracking codes, such as *TRACK*, can be used very effectively to simulate and investigate the beam behaviour in cyclotrons and especially the effect of distortions in the magnetic and/or electric field.

We would like to thank Stefan Adam for the discussions of the results and Maarten Dörenkämper for his contribution to the *TRACK* calculations.

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